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# RECENT PROGRESS IN FISSION THEORY BASED ON DIFFUSION MODEL*

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Abstract: In this talk a review on recent progress of fission theory from point of view of diffusion process is presented. The time-dependent fission rate is calculated with a simplified model for illustration. The transient phenomena are revealed. By comparison of the fission rate calculated both at saddle and scission points, the latter is emphasized for actual calculation. Some current progress on more realistic calculations related to the enhancement of the neutron emission prior to fission induced by heavy ion reactions is reported. The quantum effects which are important for low energy nuclear fission and the extension to multi-dimensional cases are also discussed. A self-consistent model to treat all the fission phenomena in a unified way, which might be also useful in the nuclear data evaluation is expected to be established in near future.

(diffusion model, fission rate, transient phenomena, saddle point, scission point, propagator)

## Introduction

Nuclear fission is one of the most important foundations for the nuclear science and technology. Although fifty years have passed since the discovery of nuclear fission, its theory is still far from completion.

The standard way in the nuclear data evaluation to calculate the fission rate (width) is to use the Bohr-Wheeler formula, which is based on the assumption of attaining instantaneous thermodynamic equilibrium between the compound-nucleus configuration and the saddle point, since nuclear fission is a dynamical process (large amplitude motion) evolving from ground-state configuration to scission point (via saddle point), the thermodynamic equilibrium theory of Bohr-Wheeler cannot fully account for the entire fission process. Thus, as early as in 1940 Kramers[1] suggested to describe the nuclear fission as a diffusion process by solving Fokker-Planck equation (FPE). However, he solved FPE only in a quasi-stationary approximation for limiting cases of small and large viscosity coefficients. The recent success of diffusion model in the studies of heavy ion reactions has revived the old suggestion of Kramers on fission problems. During the past few years, the earlier Kramers work has been re-examined and greatly extended by various authors.[2-19] The task of this talk is to give a breif report on this recent progress. We anticipate that further studies and development of such theory will eventually be useful also in nuclear data evaluation.

## The Classical Diffusion Model

### Basic Consideration and Definition

In the diffusion model of nuclear fission the motion of shape deformation of fissioning nucleus is treated as diffusion of Brownian Particle, the motion of nucleons inside the nucleus as a heat bath and the collision of nucleons with

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the nuclear wall as a classical random force. The strength of these is measured by the viscosity coefficient $\beta$ . In this model, the shape deformation coordinate $x$ and its canonical conjugate momentum P are treated as classical variables and the distribution function W(x,p) is assumed to obey a Fokker-Planck equation (FPE). In two-dimensional phase space this equation is written as

$$\frac{\partial W(x,p,t)}{\partial t} = -\frac{p}{m}\frac{\partial W(x,p,t)}{\partial x} + \frac{\partial}{\partial p}\left[(\beta p - F(x))W(x,p,t)\right] + D\frac{\partial^2 W(x,p)}{\partial p^2} \quad (1)$$

Here,

$$F(x) = -\frac{\partial}{\partial x} U(x) \quad (2)$$

relates to the potential of the fissioning system U(x), and $\beta$ is the viscosity coefficient mentioned above.The diffusion coefficient D is given by

$$D = \beta T m \quad (3)$$

where m is the inertia mass of the fissioning system and T is the nuclear temperature which is related to the excitation energy $E^*$ and the level density parameter a by the empirical formula as:

$$E^* = a T^2 \quad (4)$$

Supposed the system is initially given by a narrow distribution, which evolves with time and widens due to random collisions. In this way, the diffusion current across the saddle point $x^s$ to reach the scission point $x^{sc}$ is finally obtained and denoted as:

$$J(x^i,t) = \int_{-\infty}^{\infty} \frac{p}{m} W(x^i,p,t)\,dp \quad (5)$$

With $x^i = x^s$ or $x^{sc}$ .

Let

$$\pi(x^i,t) = \int_{-\infty}^{x^i} dx \int_{-\infty}^{\infty} W(x,p,t)\,dt \quad (6)$$

be the probability that the system is to the left of $x^i$, then the time-dependent fission rate or fission width $\Gamma_f(t)$ can be defined by

$$\Gamma_f(t) = \hbar\lambda_f(t) = \hbar J(x^i,t)/\pi(x^i,t) \quad (7)$$

For sufficiently large t, and for values of $\beta$ which are not unreasonably small, $\Gamma_f$ is expected to attain the quasi-stationary values given by kramers

$$\Gamma_f^k = \Gamma_f^{BW}\left\{\left[1+\left(\frac{\beta}{2W_0}\right)^2\right]^{\frac{1}{2}} - \frac{\beta}{2W_0}\right\} \quad (8)$$

Where $W_0$ is the frequency of the inverse harmonic oscillator potential that osculates the fission barrier at the saddle point. $\Gamma_f^{BW}$ is the one which often refers to as being given by the transition state method and can be identified with the Bohr-Wheeler expression. It is noted that the modification of the Bohr-Wheeler formula due to kramers by including a dissipation dependent factor reduces the value of the fission width for finite $\beta$ .

## Calculation of the Fission Rate

For illustration purpose, we just consider a schemetic model with a potential of the fissioning system consisting of two smoothly joined parabolas. The examples given below with parameters specified as: the inertia mass $m = \frac{A}{4} m_0$ with $m_0$ the nucleon mass, and A = 240. Assuming that m, T and $\beta$ are all independent of x, we solve the FPE to obtain the fission rates $\lambda_f(t)$ by Eq.(7) shown in Fig. 1 , for both at saddle point and scission point.

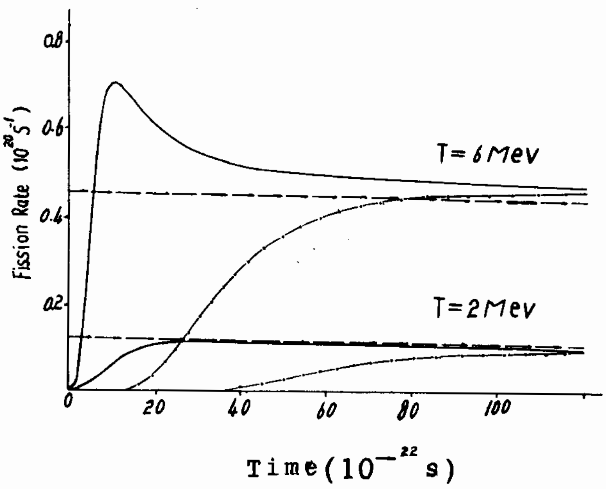

Fig. 1. The fission rate (in units of $10^{20}$ $sec^{-1}$) for $E_f$ = 4 MeV, $\beta$ = $7.5 \times 10^{21}$ $sec^{-1}$ and for two values of T as indicated versus time t (in units of $10^{-22}$ sec). The full curves are the results at saddle point ($\lambda_f^s$) and the dashed-dotted curves are the results at scission point ($\lambda_f^{sc}$). The dashed straight lines corresponds to the quasi-stationary values of kramers.

As the solution of the FPE describes the gradual spreading of the initial distribution with the probality current over the fission barrier and finally arriving at scission point, one would expect the transient process to occur. It is shown in Fig. 1., however, the transient behavior at the saddle point is quite different from that at the scission point for different ratio of $T/E_f$. At the saddle point, for $T<E_f$, $\lambda_f^s(t)$ rises gradually from 0 at t = 0 to the quasi-stationary value after a time interval $\tau$ (which is often denoted as transient time), while for $T>E_f$, $\lambda_f(t)$ increases rapidly first and decreases later to the quasi-stationary value. Thus, according to the differences in the transient behavior at the saddle point, one would expect that there exist two regimes of $T/E_f$. We call $T<E_f$ the kramers regime, in which the enhancement of the neutron emission prior to fission is anticipated and call $T>E_f$ the over shooting regime, in which the opposite effects are expected. For a realistic description, however, it is necessary to evaluate the fission rate at the scission point. It is very interesting to see in Fig. 1, that in addition to a time delay due to the system to transverse the distance from the saddle point to the scission point, the fission rate $\lambda_f^{sc}(t)$ always gradually rises to the quasi-stationary value even in case of $T>E_f$. Thus it seems more reasonable to introduce the transient time $\tau$ for this case rather than for the case at saddle point. We can define $\tau$ as a characteristic time required for the current to reach, say, 90% of the quasi-stationary value at the scission point. If $\tau$ is comparalle to the average life time $\hbar/\Gamma_n$ for neutron emission, where $\Gamma_n$ is the neutron width, one expects to find of the order of one more neutron emitted prior to fission than is calculated with a statistical model. The suppresion of the fission width within the diffusion model seems to be supported by the recent experimental measurement on average neutron multiplicity[20].

## Average Neutron Multiplicity

We show here an example[15] based on a more realistic calculation on neutron multiplicities prior to fission, in the case of the reaction $^{16}O + ^{142}Nd \rightarrow ^{158}Er$ at 207 MeV. It is shown in

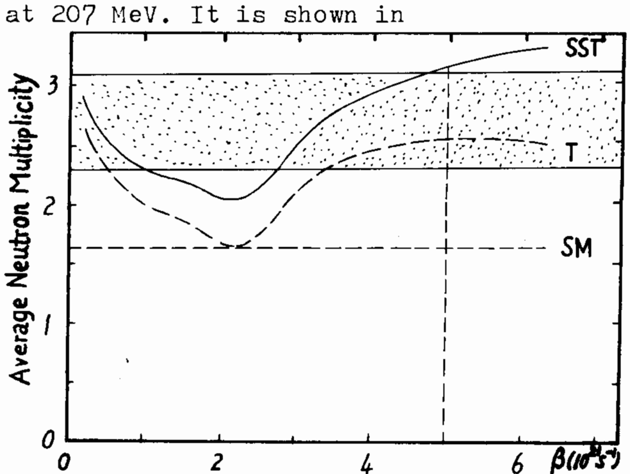


Fig. 2. Average neutron multiplicity $\langle\nu\rangle$ versus the reduced dissipation coefficient $\beta$ . The curves labelled SM,T, and SST refer to the statistical model, the inclusion of transients (defined at saddle point) and the mean saddle-to-scission time, respectively. The experimental result[20] is given by the shaded band, whose upper intersection with the solid curve determines the upper limit of $\beta$ indicated by the vertical dashed line.

Fig. 2, that the enhancement of the neutrons emitted prior to fission can be reasonably understood in terms of neutron emission during the transient time required to build up the quasi-stationary current over the barrier and the mean time required for the system to descend from the saddle point to scission point.

These authors even have determined the limit $\beta \lesssim 5\times10^{21}$ $sec^{-1}$ for the reduced nuclear dissipation coefficient. This quantitative conclusion, however, should not be taken too seriouly and further more systematic studies and improvement are needed.

## Quantum Effects in Diffusion Model

The diffusion model discussed in previous section is purely classical one which is applicable only for describing the fission process at high temperature. In order to extend the classical diffusion model to low temperature regime the quantum effects should be included. For this purpose a simple model of quantum system has been established[17] to reproduce the classical FPE at high temperalure and to retain the quantum effects at low temperature case. This model Hamiltonian has three parts:

$$H = H_A + H_B + H_I \qquad (9)$$

Where $H_A$ is the Hamiltonian of the subsystem A considered in the problem, that is the quantum Brownian motion in the fission potential. $H_B$ is the Hamiltonian of "heat bath" consisting of N harmonic oscillators, and $H_I$ is a linear coupling term between the subsystems A and B with a coupling strength $C_k$ (k runs from 1 to N). In order to make a connection between this microscopic coupling strength with the macroscopic viscosity and to reproduce the FPE at high temperature, the distribution density of oscillator frequencies for subsystem B is introduced as done, for instance, in solid state physics[21].

$$\rho_D(W)C^2(W) = \begin{cases} 2m_b\eta W^2/\pi & W<\Omega \\ 0 & W>\Omega \end{cases} \qquad (10)$$

Here, $\eta$ is the viscosity coefficient, $\rho_D$ the distribution density of oscillators in subsystem B with a high frequency Cut-off $\Omega$. According to formula (10), a discrete distribution of oscillator frequencies is replaced by a continuous one. This replacement is very crucial in a sense that leads the microscopically reversible process to the macroscopically irreversible one. Hence, the subsystem B has become a real heat bath now.

So far the potential in the subsystem A is general. In the investigation of the diffusion process it has already been proved that the locally harmonic approximation can be used to solve the FPE for real fission potential quite well except for very small viscosity[16]. It seems resonable to extend this approach to the quantum case. In this approximation, the fission potential can be divided into many small regions, each of them can be approximated by a harmonic oscillator with the frequencies depending on collective coordinate x. Based on this simplified model by means of Feynman-Vernon theory an analytical expression for the reduced density operator can be obtained for each small region of the fission potential. The final reduced density operator for the whole fission potential can easily be carried out numerically as done in the classical case within the locally harmonic approximation.

The calculated results of the fission rates at the saddle point are illustrated in Fig. 3 for both quantum (solid curves) and classical (dashed curves) cases at temperature of T = 0.5 and 8 MeV. It is seen that at T = 8 MeV,

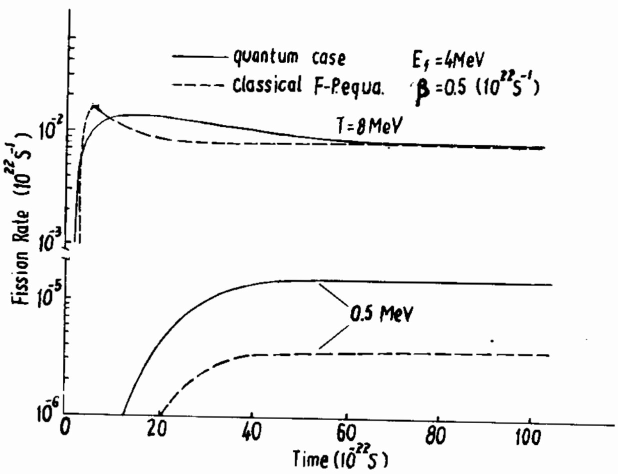

Fig. 3. The fission rates for both quantum and classical cases are versus time t. All results are calculated for a fission barrier $E_f$ = 4 MeV and a reduced viscosity coefficient $\beta = 5\times10^{21}$ $sec^{-1}$.

the results of both quantum and classical approaches are very similar. This implies that the classical diffusion is the limiting case of the quantum diffusion at high temperature. In the lower part of Fig. 3, the results for low temperature T = 0.5 MeV are shown. Because of the enhancement of the diffusion due to quantum effects the fission rate in quantum case is considerably larger than that in classical case in low temperature region as expected. This model seems to have improved the behavior of the classical diffusion at low temperature by including the quantum effects and kept its reasonable features at high temperature. Further work concerning the its link with the microscopic foundation and comparison with experiments is very much encouraged.

## Multi-dimensional Case

Generally speaking, for a description of the shape deformations occuring during the fission process several degrees of freedom are necessary. In this respect, the induced fission is described in terms of a FPE for N degrees of freedom. Owing to the difficulty in solving multi-dimensional FPE in a general way, progress has only been made in some specific cases with various approximations.

## Stationary Diffusion Over a Multi-dimensional Fission Barrier

The Kramers' formula has been generalized to stationary diffusion over a potential barrier for N degrees of freedom.[2,9,14] From a calculation of the two-dimensional case compared with that of one-dimensional case, it is found that the deviation is not significant within a reasonable range of parameters[14]. Thus, it will not be described in details here.

## Fission-Fragment Kinetic Energy and Mass Distributions

In order to descrobe the kinetic energy or mass distribution of the fission fragments more than two degrees of freedom for shape deformations are necessary. Recently several authors[18,19] have performed this kind of calculation based on (C,h,$\alpha$) parameterization[21]. Two deformation parameters describe symmetric shape that is C and h which describe the elongation and necking, respectively. The parameter $\alpha$ characterizes the left-right asymmetry of the mass. In actual calculation, instead of C, the parameter $\rho$ is used as an elongation coordinate which is defined as the half of the distance between the centers of mass of the two future fragments.

The variances of mass distribution of fission fragments from a number of the excited nuclei (the temperature of these nuclei at the saddle point $T_S \approx 1.5$ MeV) have been calculated based on FPE[18] which are shown in Fig. 4 (dased curves).

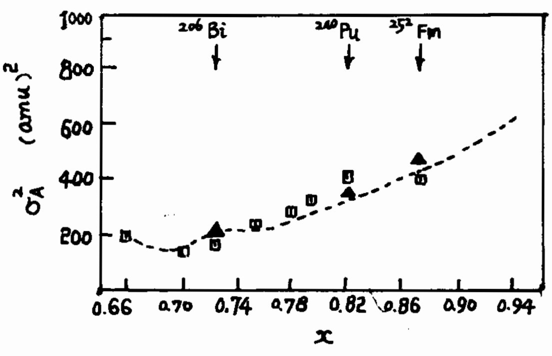


Fig. 4. The variances of mass distribution versus fissibility parameter x. The squares correspond to experimental values.[22,23] The dashed curves represent the calculated results taken from Ref.18, while the sull triangles are also the thcoretical values taken from Ref. 19,.

Some preliminary results[19] calculated with Langevin equation are also shown there (full triangles).

It is seen that the calculations can reproduce the experimentally observed increase of the mass variance with of growth of the fissibility parameter x. This increase of the calculated variances is mainly due to a considerable decrease in the stiffness coefficients corresponding to the saddle points for heavy fissioning nuclei and to the non-equilibrium character of the decent from saddle to scission.

The variances of the kinetic energy distribution calculated both with FPE[18] and Langevin equation[19] are shown in Fig. 5.

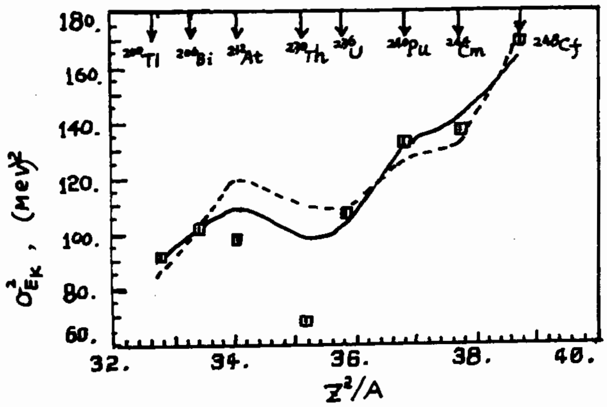


Fig. 5. The variance of the kinetic energy distribution versus parameter $Z^2/A$. The squares represent the experimental data[24], while the dashed and solid curved correspond to the calculations based on FPE[18] and on Langevin equation[19], respectively.

In the calculation, the fluctuations in collective coordinates near scission and prescission kinetic energy have been taken into account simultaneously. Owing to the fact that heavy fissioning nucleus can attain greater elongation without neck rupture than the light one, the fluctuation in collective coordinate rapidly increases with $Z^2/A$. As a result the variances increase with $Z^2/A$ are understandable, since the fluctuations in collective coordinates contribute the main part to the variances. The calculations again are in agreement with the experimental data.

Regarding the extension to multi-dimensional calculation based on diffusion model, it seems the Langevin equation which is equivalent to FPE has the advantage in saving the computer time and is more convenient and feasible to compute for cases with coordinate dependent parameters by Monte-Carlo simulation method.

## Conclusion

The recent progress in nuclear fission theory based on diffusion model has been breifly reported. The Basic assumption of this model lies in the fact that the single-particle degrees of freedom are equilibrated ($\sim 10^{-22}$ sec) much faster than the relaxation time of deformation shape ($10^{-21}$ sec).

The standard statistical and dynamical models are two limiting cases of diffusion model.

The results obtained so far with this model are encouraging. This provides us with a good basis for further studies.

Up to now only the classical one-dimensional problems have been investigated rather thoroughly, further studies on quantum effects and extension to nulti-dimensional cases are needed.

It is hopeful that whithin a few years we can eventually establish a self-consistent theory to treat all fission problems in an unified way, which might also be useful in nuclear data evaluation.

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